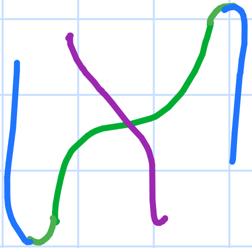
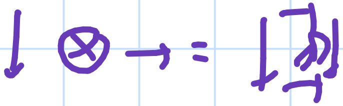




∞-categories w/ adjoints

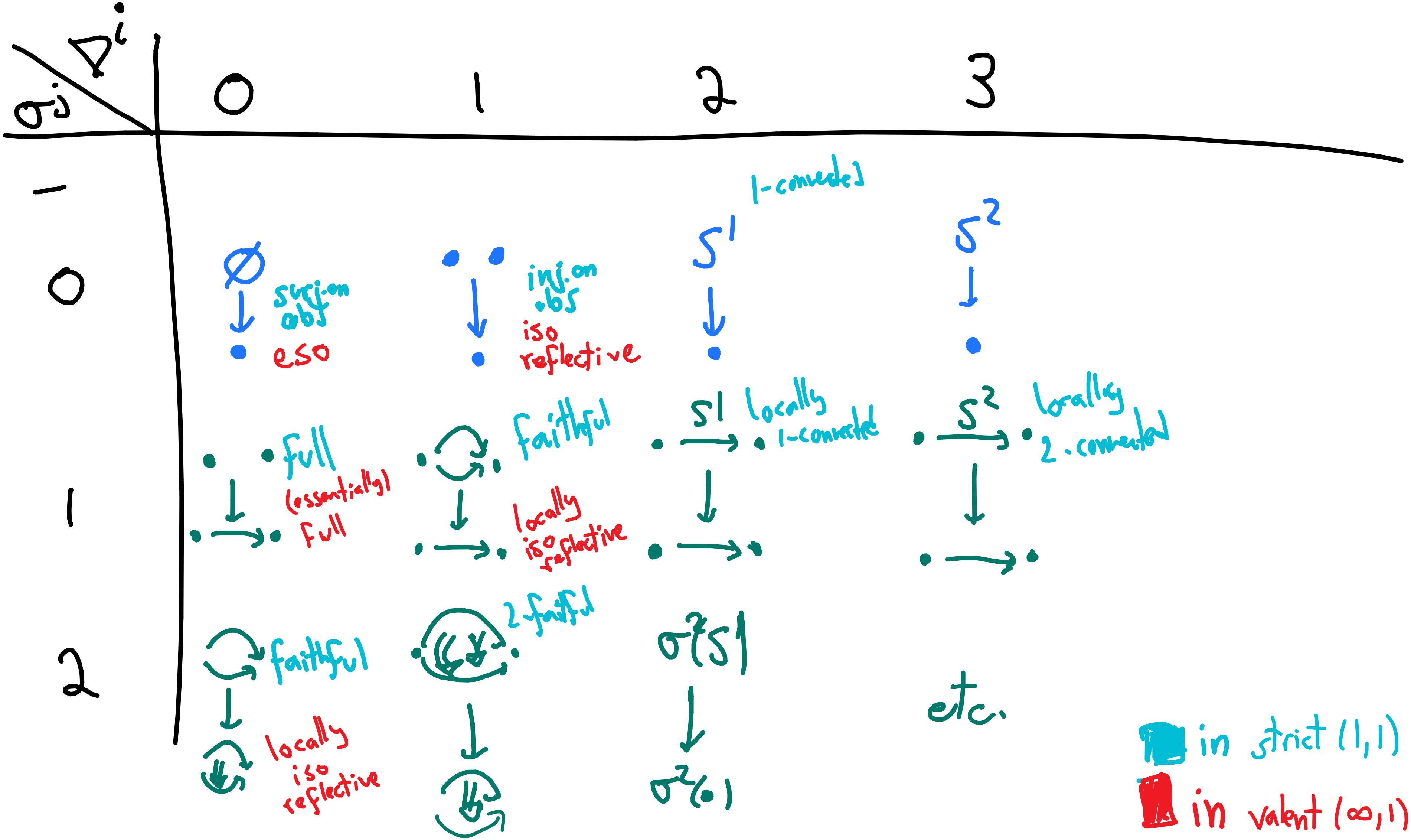
are a bit like spaces

↳ thinking cellularly

Daniel Teixeira, Dalhousie University

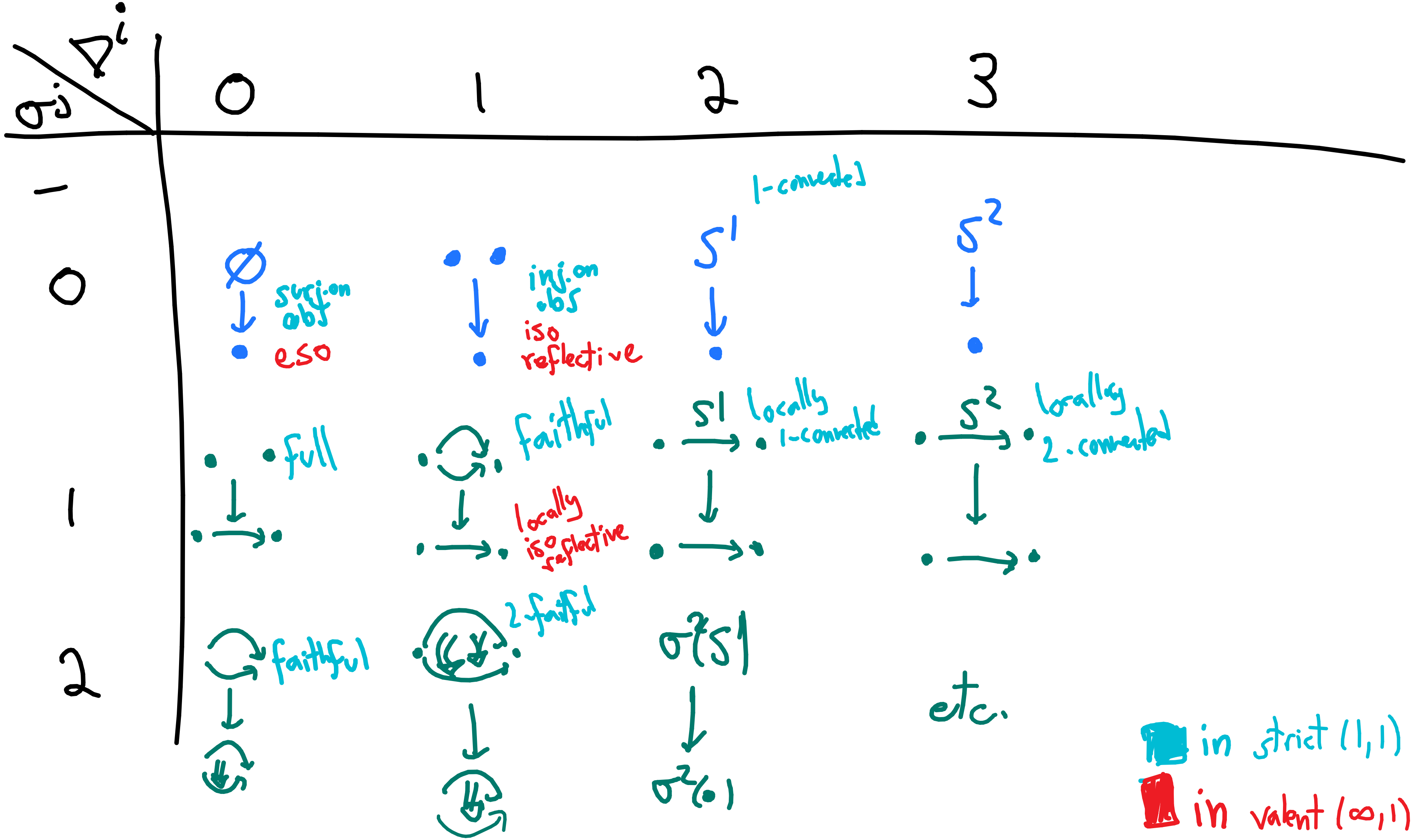
FMCS 2026

$$\cdots f^{LL} \dashv f^{L} \dashv f \dashv f^{R} \dashv f^{RR} \dashv \cdots$$

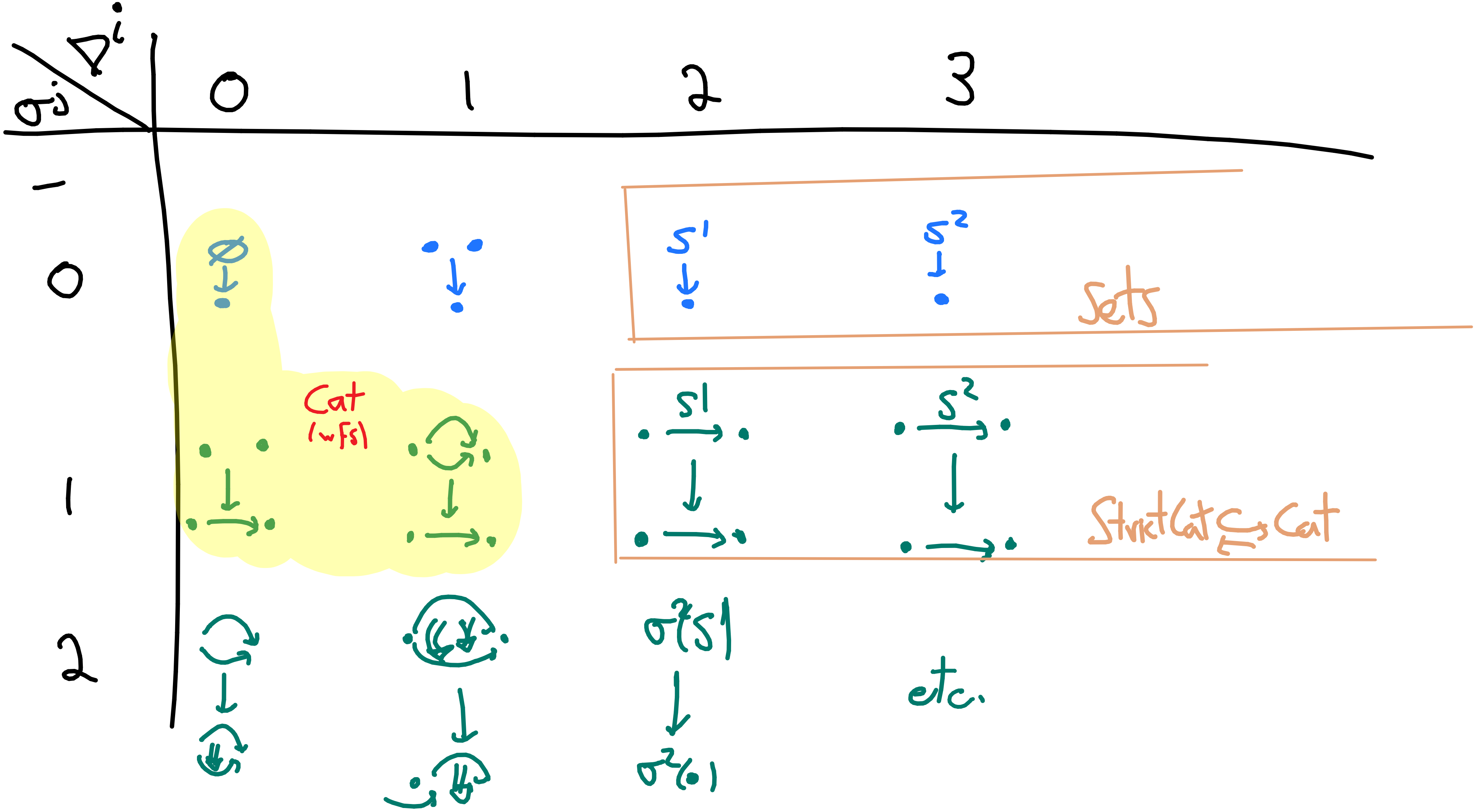


σ^j
Δ^i
0
1
2
3
−
0
1
2
1-connected
S^1
S^2
∅
surj. on obj
eso
inj. on obj
iso reflective
full
(essentially) full
faithful
locally iso reflective
S^1
locally 1-connected
S^2
locally 2-connected
faithful
locally iso reflective
2-faithful
σ^2(S^1)
σ^2(•)
etc.
in strict (1,1)
in valent (∞,1)


Always has been
wait, every localization is reflective?

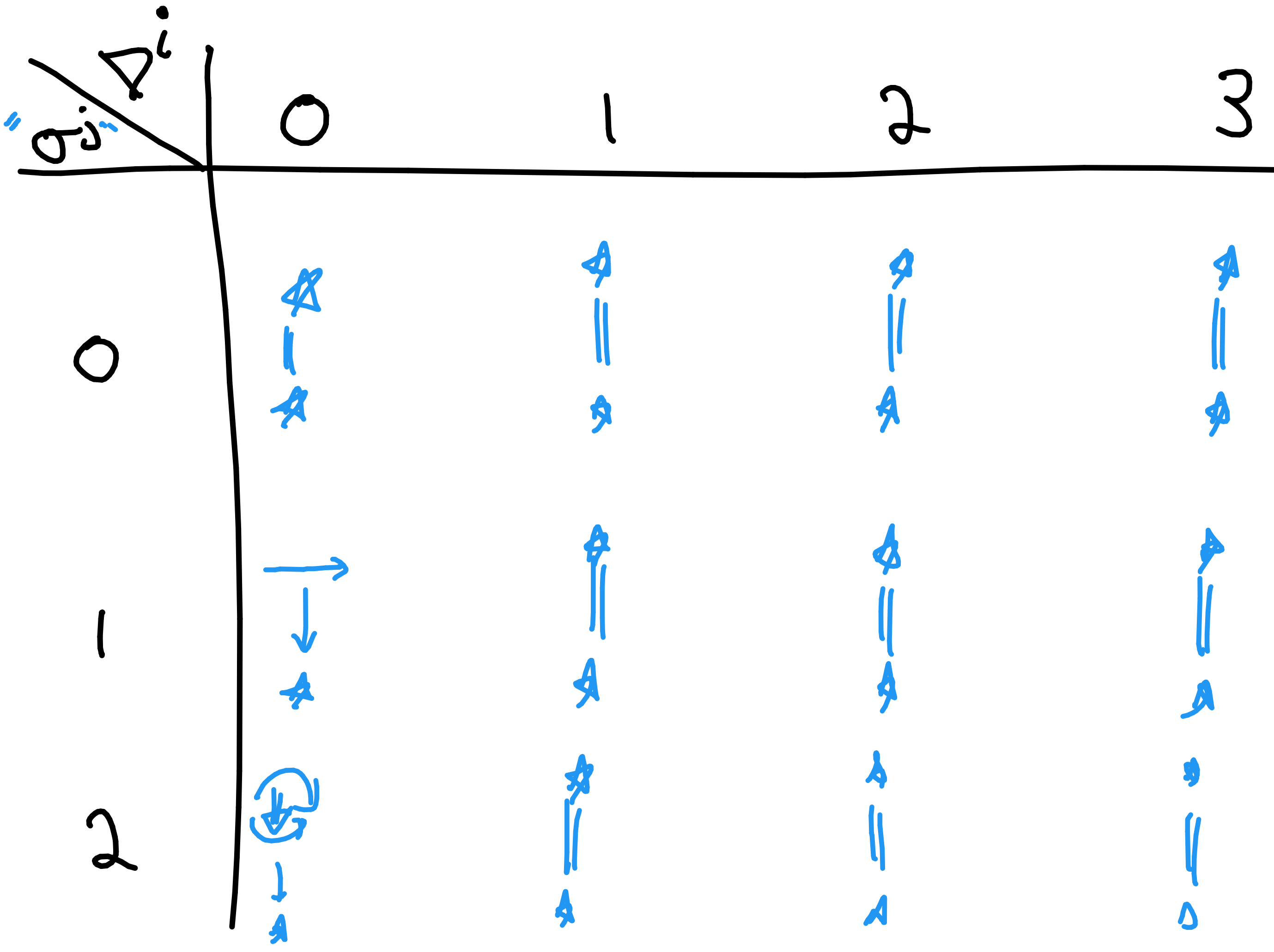


Δ^i
σ^j
0
1
2
3
-
0
1
2
$\varnothing$
surj. on obj
eso
inj. on objs
iso
reflective
S^1
1-connected
S^2
full
faithful
S^1
locally 1-connected
S^2
locally 2-connected
locally iso reflective
faithful
2-faithful
$\sigma^2(S^1)$
$\sigma^2(\bullet)$
etc.
in strict (1,1)
in valent $(\infty,1)$

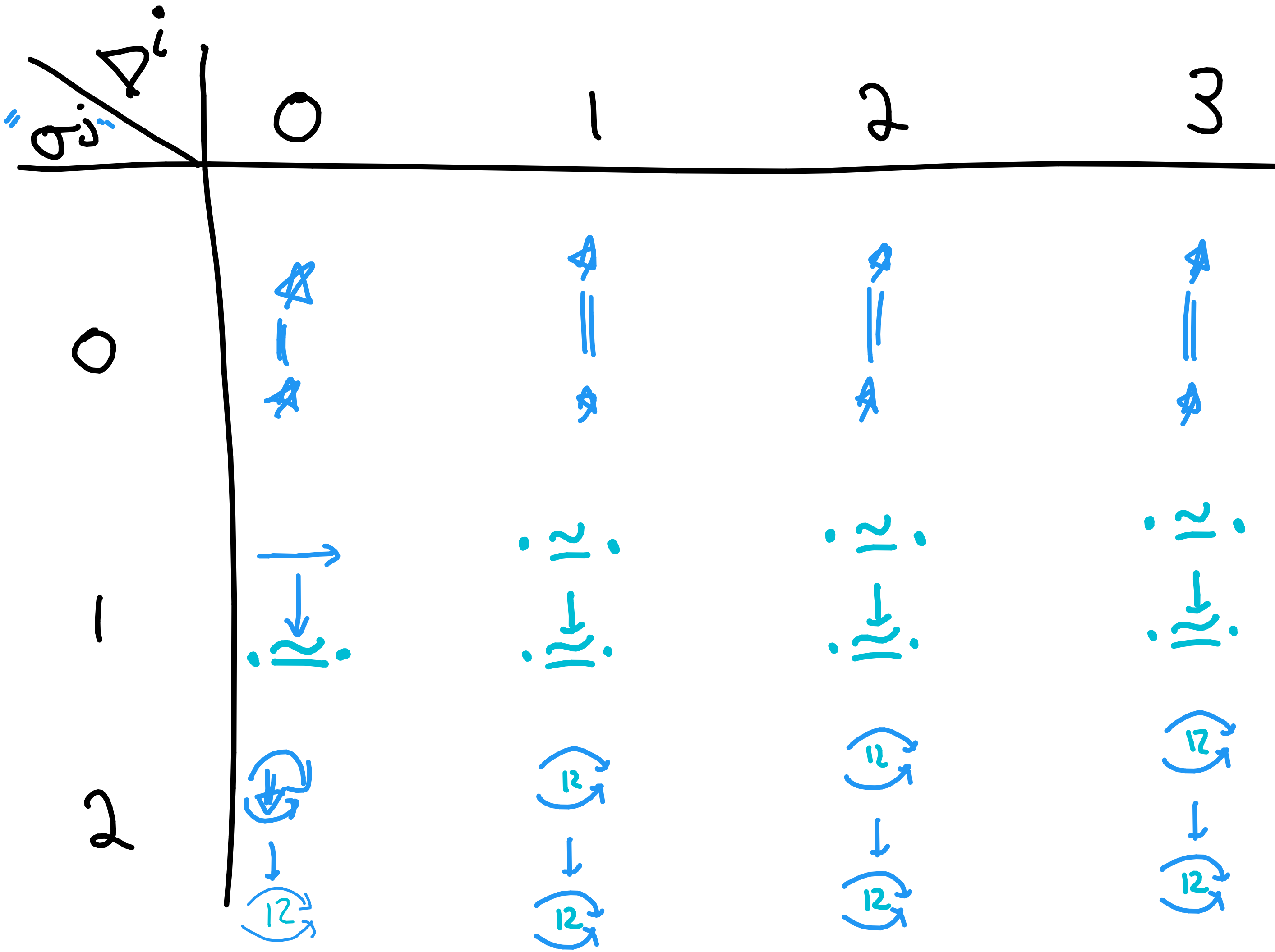

σj
Δi
0
1
2
3
-1
0
1
2
∅
S1
S2
Sets
Cat
(wFs)
S1
S2
StrictCat ⊆ Cat
σ2(S1)
σ2(•)
etc.

$\sigma^j \backslash \Delta^i$	0	1	2	3
0	N/A	N/A	N/A	N/A
1	$\begin{array}{c} \bullet \xrightarrow{\sim} \bullet \\ \downarrow \\ \bullet \end{array}$	$\begin{array}{c} \bullet \\ \parallel \\ \bullet \end{array}$	$\begin{array}{c} \bullet \\ \parallel \\ \bullet \end{array}$	$\begin{array}{c} \bullet \\ \parallel \\ \bullet \end{array}$
2	$\begin{array}{c} \bullet \underset{}{\overset{}{\rightrightarrows}}_{\Downarrow 12} \bullet \\ \downarrow \\ \bullet \longrightarrow \bullet \end{array}$	$\begin{array}{c} \bullet \longrightarrow \bullet \\ \parallel \\ \bullet \longrightarrow \bullet \end{array}$	$\begin{array}{c} \bullet \longrightarrow \bullet \\ \parallel \\ \bullet \longrightarrow \bullet \end{array}$	$\begin{array}{c} \bullet \longrightarrow \bullet \\ \parallel \\ \bullet \longrightarrow \bullet \end{array}$

For $\infty\mathrm{Cat}^{uni}$

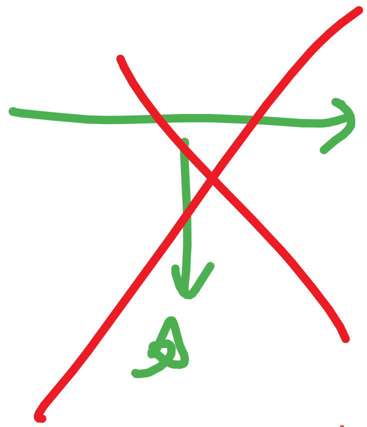


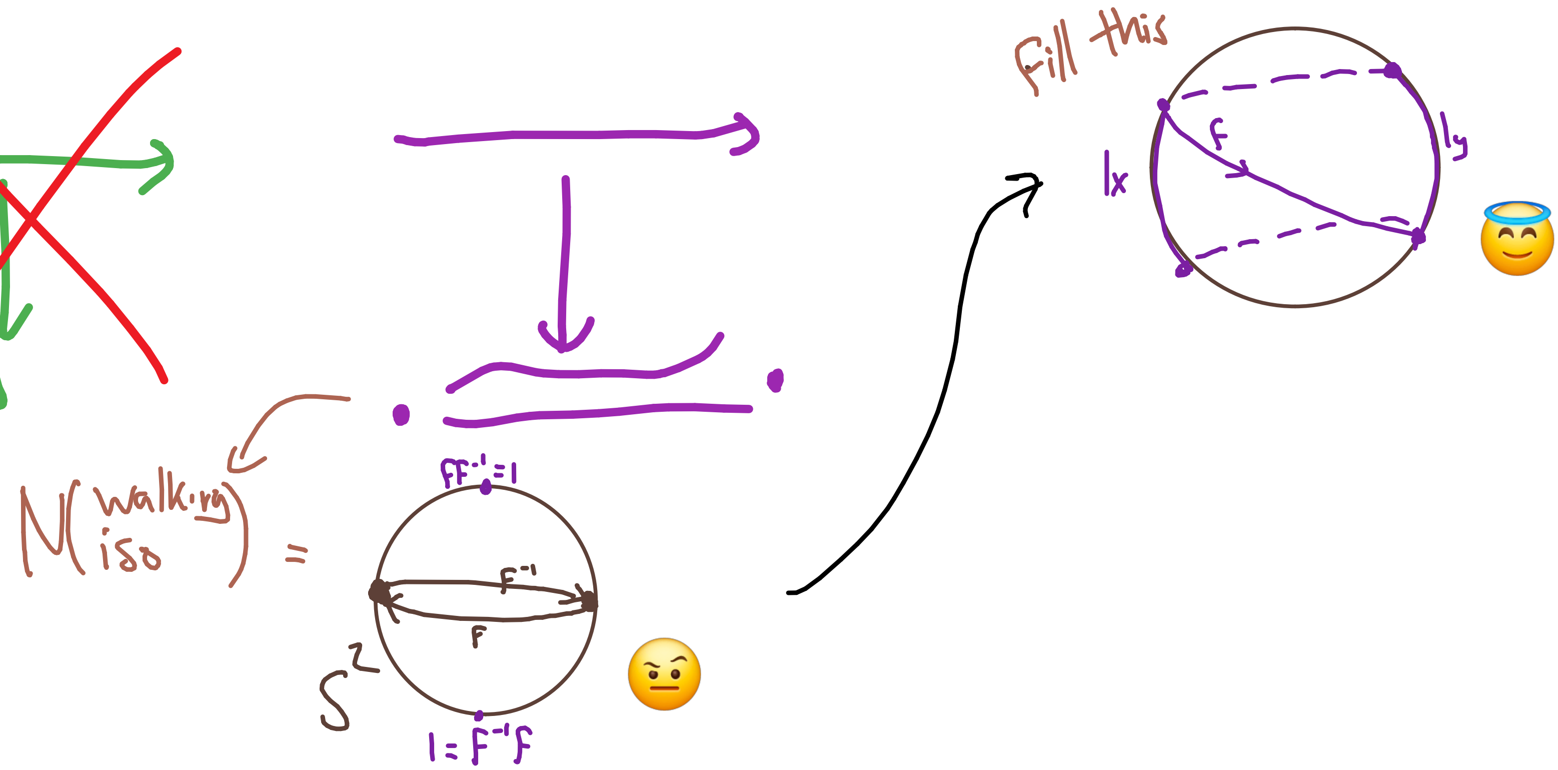
For $n\mathrm{Cat}^{\mathrm{univ}}$

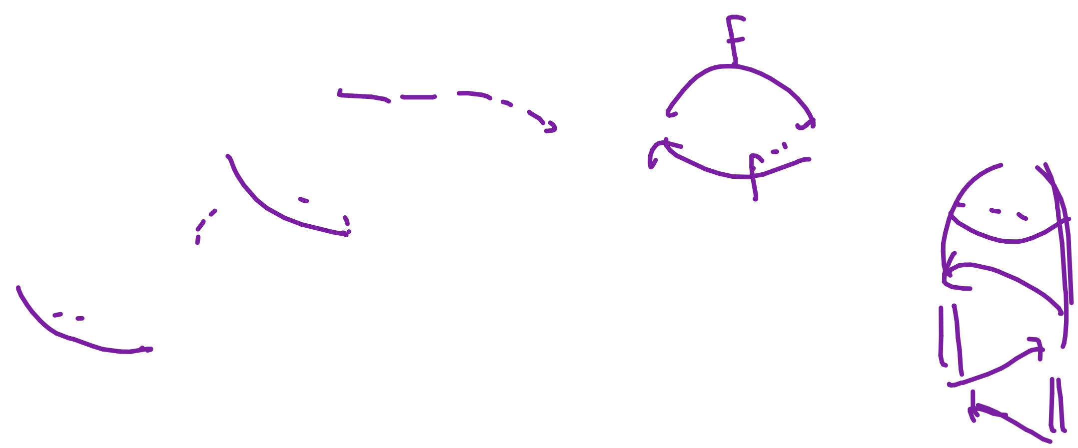


For $n\mathrm{Cat}^{\text{valent}}$

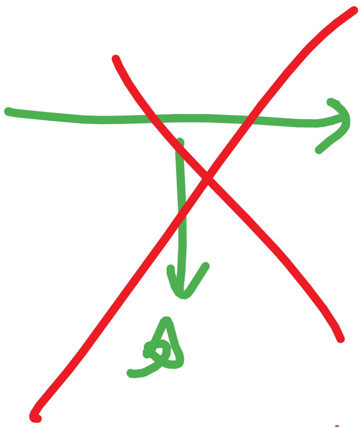
What if we groupoidified BEFORE univalentizing?

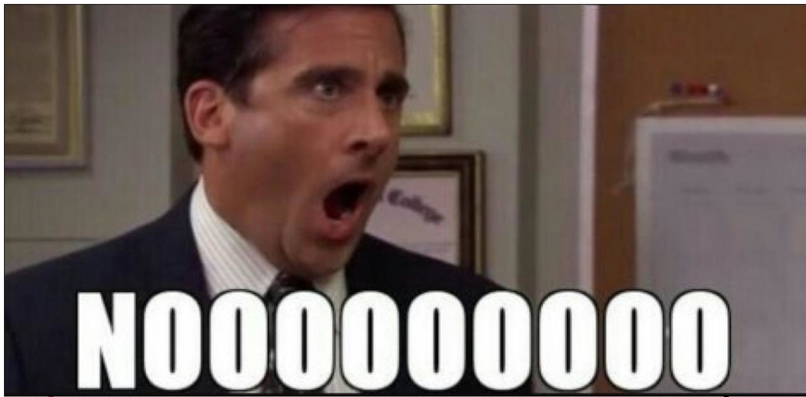




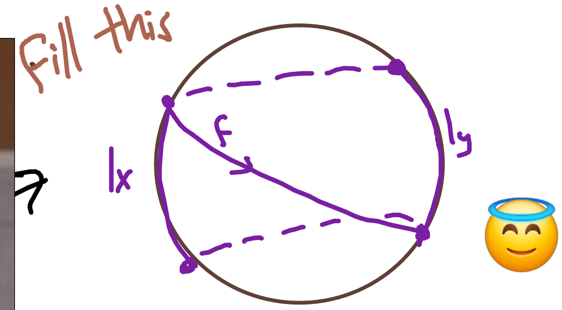


What if we groupoidified BEFORE univalentizing?

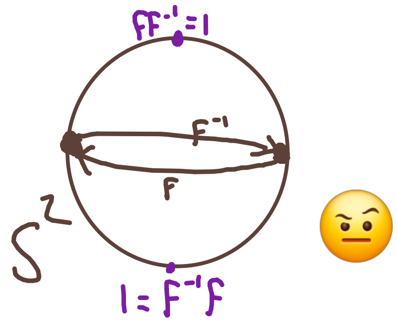




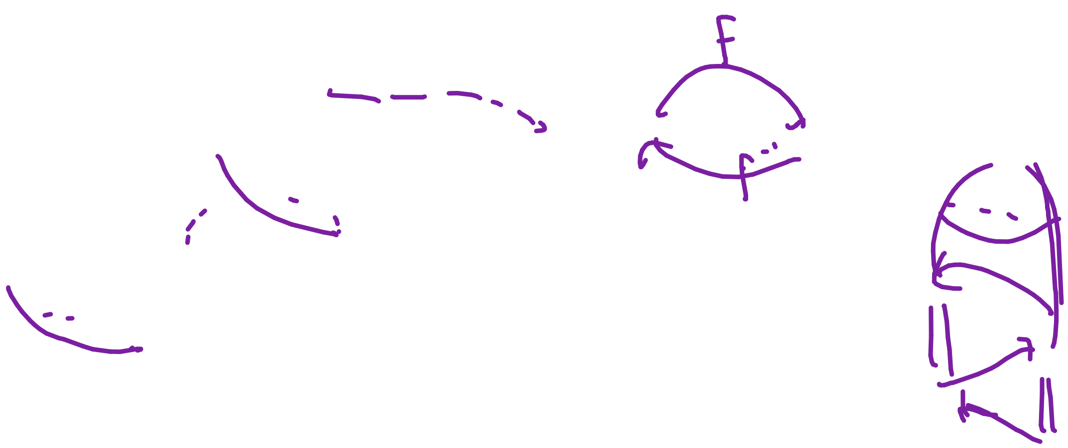
Fill this



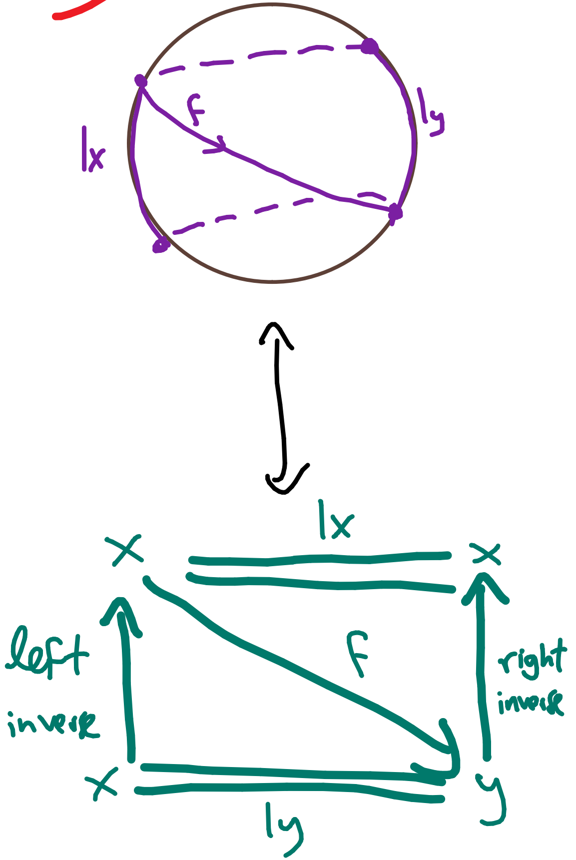
$N\begin{pmatrix}\text{walking}\\ \text{iso}\end{pmatrix} =$



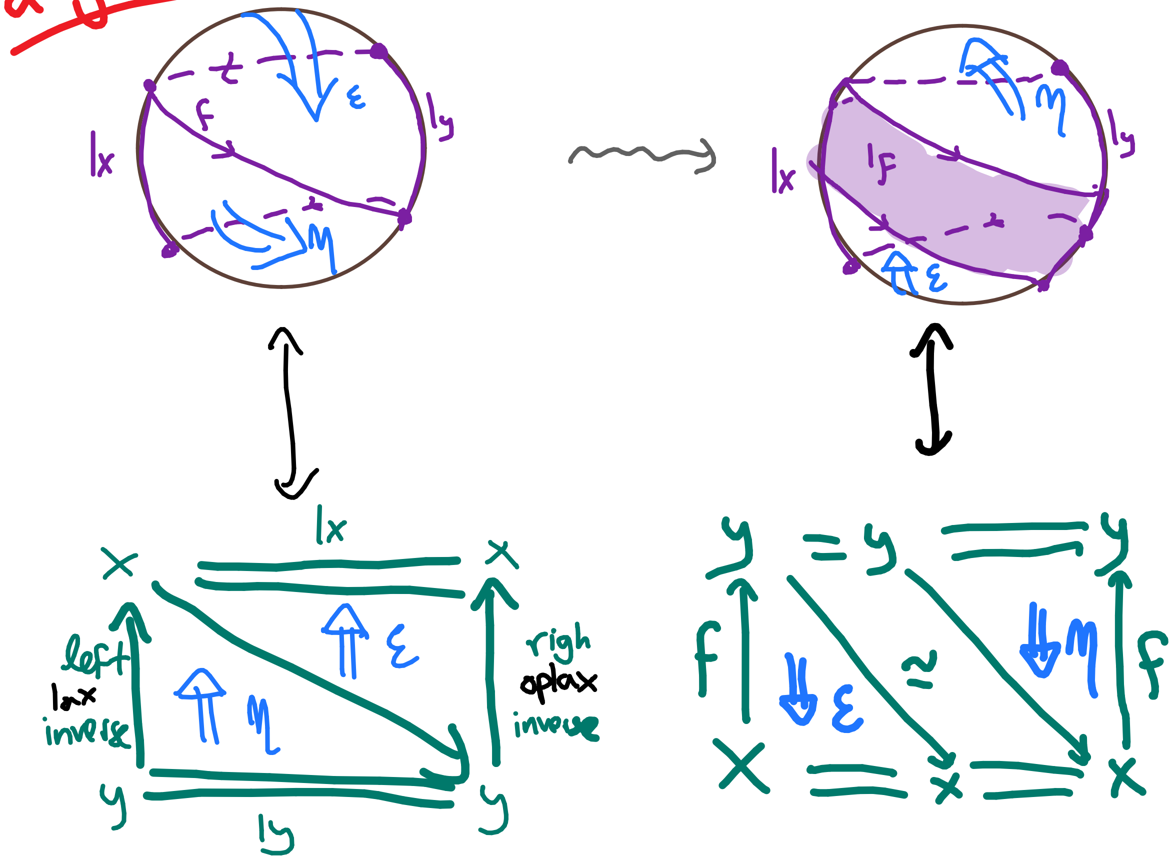
Fill this too



isos



adjunctions



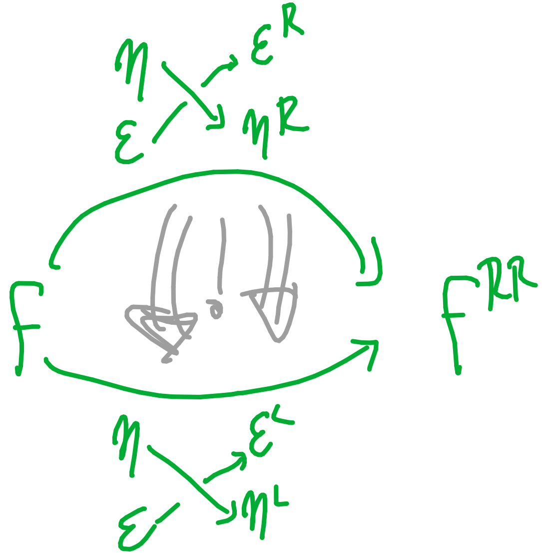
All adjunctions

Goal: study

$\dashv \dots f^{LL} \dashv f^{L} \dashv f \dashv f^{R} \dashv f^{RR} \dots$

$\dots \eta^{L} \dashv \eta \dashv \eta^{R} \dots$

$\dots \varepsilon^{L} \dashv \varepsilon \dashv \varepsilon^{R} \dots$



Fact: adjunction $f \dashv f^R$ via

$$\begin{cases} \eta : 1 \to f^R f \\ \varepsilon : f f^R \to 1 \end{cases}$$

$+$

$$\begin{cases} \eta \dashv \eta^R : f^R f \to 1 \\ \varepsilon \dashv \varepsilon^R \end{cases}$$

OR

$$\begin{cases} \eta^L \dashv \eta \\ \varepsilon^L \dashv \eta \end{cases}$$

$\rightsquigarrow$ $f^R \dashv f$ via (ε^R, η^R)

OR

$f^R \dashv f$ via (ε^L, η^L)

i.e.

$\mathrm{op}, 2\mathrm{op} \circlearrowright 2\mathrm{Cat}$ trivializes on $2\mathrm{Cat}^{\mathrm{adj}}$ everyone is a fixed point

$\mathbb{Z}/2$

NON CANONICALLY

$1\mathrm{op}, 2\mathrm{op} \circlearrowright 2\mathrm{Cat}$ seems to trivialize on $\infty\mathrm{Cat}^{\mathrm{adj}}$

$\mathbb{Z}/2$

CANONICALLY

HOPES

- build ∞-categories w/ adjoints cellularly ($\xrightarrow[\text{adj}]{\text{full}}$ instead of $\xrightarrow{\text{iso}}$ or $\to$)

! - mimic htpy theory constructions

↳ hope: this is much better than Cat_∞

- relation to high dim. string diagrams
- nice Postnikov theory for $\infty\mathrm{CatAdj}$ → main object of my thesis
- unstable version of the cobordism hypothesis

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TY!